CHUNG CHENG HIGH SCHOOL (YISHUN)
MID-YEAR EXAMINATION 200 8

SECONDARY TWO EXPRESS

MATHEMATICS

Duration: **2 hours** Date: 06-05-2008

Name: _____ ( ) Class: _____

Instructions:

1 Answer **All** questions in this Paper

2 All essential workings must be clearly shown in the spaces provided below the question. Omission of essential working will result in loss of marks

3 You are allowed to use calculator in this Paper

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at each question or part question
- The total marks for this paper is 100
- You should not spend too much time on any one question

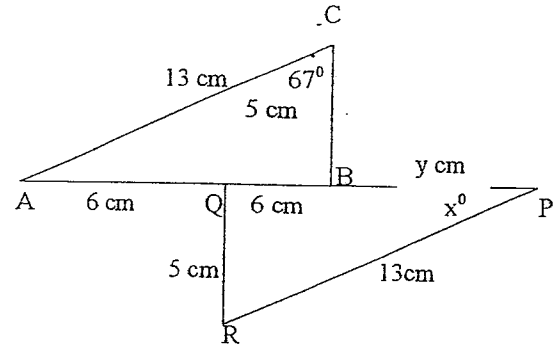
For examiner's use
/ 100

This paper consists of 10 printed pages including this cover page

[Turn over

40

1 Given that ΔABC and ΔPQR are congruent. Calculate x and y .



Ans: x^0 = _____ [1]

y cm = _____ [1]

2 The length (L) stretched by a spring varies directly as the amount of force (F) applied. A force of 20 kg stretches the spring by 15 cm. How long would the spring be stretched by a force of 24 kg ?.

Ans: _____ [2]

3 A road of length 8 km is represented on a map by a line of 5 cm.
Calculate,

(a) the scale of the map in the form 1 : n ,

(b) the actual area of a plot of land, in square kilometers , that is represented by an area of 12.5 cm^2 on the map.

Ans:(a)_____ [1]

(b)_____ [2]

4 The time taken, t minutes , to download a file from a computer is inversely proportional to the internet connection speed, v kB/s. When the speed is 64 kB/s , the time taken is 12 minutes.

(a) Find the formula connecting t and v .

(b) Find the time taken when v = 512 kB/s .

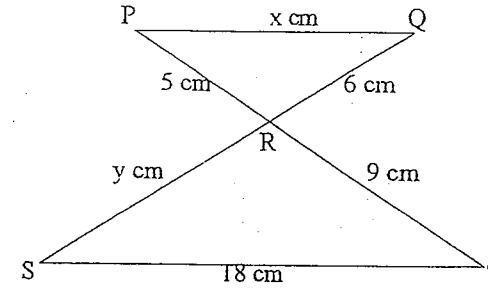
Ans: (a)_____ [3]

(b) _____ [1]

5 In the figure below , ΔPQR is similar to ΔTSR . Given that PR=5 cm , QR=6 cm , ST= 18 cm and TR= 9 cm , calculate the length

(i) PQ ,

(ii) SR .



Ans: (i) _____ [2]

(ii)_____ [2]

6 Expand each of the following:

(a) $(3x-5y)^2$ (b) $(2x+y)(x-5y)$

(c) $(2x-5y)(3x+y)+5y(4x+y)$

Ans: (a)_____ [2]

(b)_____ [2]

(c)_____ [2]

7 Factorise the following expressions completely

(a) $144x^2-25y^2$

(b) $5x^2-11x-12$

Ans: (a)_____ [2]

(b)_____ [2]

8 Evaluate the following by factorization without using calculators.

(a) $267^2 - 233^2$

(b) $32.6 \times 37 + 63 \times 32.6$

Ans: (a)_____ [2]

(b)_____ [2]

9 Given that $x^2 - y^2 = 64$ and $x + y = 4$. Calculate the values of

(a) $x - y$,

(b) $(2x - 2y)^2$.

Ans: (a)_____ [2]

(b)_____ [2]

10 Simplify each of the following expressions:

(a) $\dfrac{(7x^2y)^2 z^4}{21yz}$ (b) $\dfrac{8x^2 - 16xy}{5x - 10y}$

Ans: (a)_____ [2]

(b)_____ [2]

11 Simplify each of the following expressions:

(a) $\dfrac{x^2 - 25}{5 - x}$ (b) $\dfrac{4x^2 - y^2}{12x^2 - 4xy - y^2}$

Ans: (a)_____ [2]

(b)_____ [3]

12 Simplify each of the following , giving your answers in the lowest terms.

(a) $\frac{6q^2}{21rp^2} x \frac{3pr^3}{2q} \div \frac{3r^2}{7pq}$

(b) $\frac{(p+2)(p+1)}{p+4} \div \frac{2p^2+p-1}{(2p-1)(p+4)}$

Ans: (a)_____ [2]

(b)_____ [3]

13 Simplify each of the following as a single fraction.

(a) $\frac{2}{(x-3)} - \frac{1}{(2x-5)}$

(b) $\frac{3}{2(x-1)} - \frac{x+2}{(x^2-1)}$

Ans: (a)_____ [2]

(b)_____ [3]

14 Given that $x = \frac{5y+3}{y+5}$, express y in terms of x .

Ans: _____ [3]

15 Given that $s = r^2 + rl$, make l the subject of the formula. Taking $\pi = 22/7$, find the value of l when $s = 200$ and $r = 5$. Give your answer correct to 3 significant figures.

Ans: (formula) $l =$ _____ [2]

$l =$ _____ [2]

16 Solve the following pair of simultaneous equations using the elimination or substitution method.
$3x - 4y = 10$, $5x + 7y = 3$

Ans: $x =$ _____

$y =$ _____ [5]

17 Complete the following table of values for x and y for the equation $3x - 4y = 10$. [2]

x	- 2	0	4
y			

(a) Using a scale of 2 cm to 1 unit on both axes, plot the points and draw a straight line through the points. [3]

(b) From the graph, find

(i) the value of y when $x = -1$, [1]

(ii) the value of x when $y = -2$. [1]

18 (a) Solve the equation $2x^2 - 9x = 0$

Ans: (a)_____ [3]

(b) The equation $2x^2 + kx - 15 = 0$ has a solution of $x = 3$.

(i) Find k.

(ii) Find the other x value

Ans: (b)(i)_____ [2]

(b)(ii)_____ [2]

19 Solve the equation $4(x-3)(x+1) = 5(x-3)$

Ans: _____ [4]

20 A housewife finds that 5 cans of condensed milk and 3 jars of instant coffee cost \$27 while 12 cans of condensed milk and 5 jars of instant coffee cost \$49.40. Assume that the cost of each can of condensed milk is \$x and each jar of instant coffee is \$y , form a pair of simultaneous equations and hence find the total cost for 7 cans of condensed milk and 2 jars of instant coffee.

[7]

21 **Answer the whole of this question on a sheet of graph paper.**

The following is a table showing corresponding values of x and y which are related by the equation $y = -x^2 + 2x + 3$.

x	-3	-2	-1	0	1	2	3	4	5
y	-12	a	0	3	4	b	0	-5	-12

(a) Calculate the values of a and b . [2]

(b) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 2 units on the y-axis , draw the graph of $y = -x^2 + 2x + 3$ for $-3 \leq x \leq 5$. [4]

(c) Use your graph to find

(i) the value of y when $x = 1.8$, [1]

(ii) the values of x when $y = -2$, [2]

(iii) the equation of the line of symmetry for $y = -x^2 + 2x + 3$, [1]

(iv) the maximum value of y . [1]

END OF PAPER

CHUNG CHENG HIGH SCHOOL (YISHUN)
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MATHEMATICS

Marking Scheme

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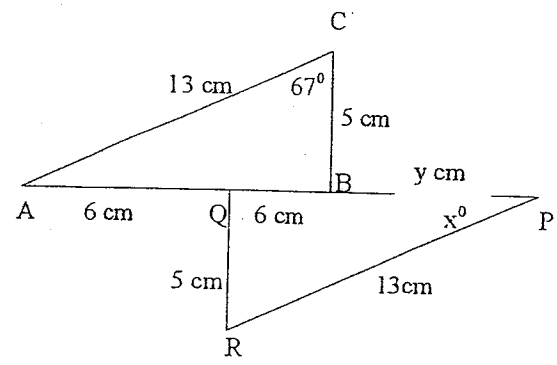
Sec 2 Express Mathematics (Mid-Year Exam 2008) **Marking Scheme**

1 Given that ΔABC and ΔPQR are congruent. Calculate x and y .

[Solution]



$\Delta ABC \equiv \Delta PQR$ (Given)

$PQ = y + 3$

$PQ = AB$ (corr. sides of $\equiv \Delta$)

$y + 3 = 6 + 3$

$y = 6$ cm -------A1

angle PRQ = angle ACB (corr. angles of $\equiv \Delta$)

$= 67^0$

$x^0 + 67^0 + 90^0 = 180^0$

$x^0 = 23^0$ --------A1

Ans: x^0 = 23^0 [1]

y cm = 6 cm [1]

2 The length (L) stretched by a spring varies directly as the amount of force (F) applied. A force of 20 kg stretches the spring by 15 cm. How long would the spring be stretched by a force of 24 kg ?

[Solution]

Let L be the length stretched by the spring and F be the force applied.

$L = kF$ where k = constant of variation

$15 = k\,(20) \Rightarrow k =$ ------- M1 $\Rightarrow L =$ F

when F = 24 kg ,

$L =$ $(24) \Rightarrow L = 18$ cm ----------A1

Ans: 18 cm [2]

3 A road of length 8 km is represented on a map by a line of 5 cm.

Calculate,

(a) the scale of the map in the form 1 : n ,

(b) the actual area of a plot of land, in square kilometers , that is represented by an area of 12.5 cm^2 on the map.

[Solution]

(a)

5 cm represent 8 km or 800 000 cm

1 cm represents 160 000 cm $\Rightarrow$ 1 : 160 000 ---------- A1

(b) 5 cm represent 8 km $\Rightarrow$ 1 cm represents 8/5 km $\Rightarrow$ 1 cm^2 represents $(8/5)^2$ km^2 -------- M1

12.5 cm^2 represent $12.5 \times (8/5)^2 \Rightarrow 32$ km^2 ----------A1

Ans:(a) 1:160 000 [1]

(b) 32 km^2 [2]

4 The time taken, t minutes , to download a file from a computer is inversely proportional to the internet connection speed, v kB/s. When the speed is 64 kB/s , the time taken is 12 minutes.

(a) Find the formula connecting t and v .

(b) Find the time taken when v = 512 kB/s. [4]

[Solution]

t is inversely proportional to v => t = k (1/v) ---------- M1 (where k is the constant)

when t = 12 minutes and v = 64 kB/s ,

=> 12 = k / 64 => k = 12x64 --------- B1 => t = (12 x 64) / v --------- M1

When v = 512 kB/s , t = (12x64) / 512

t = 1.5 minutes ---------- A1

Ans: (a) _____ [3]

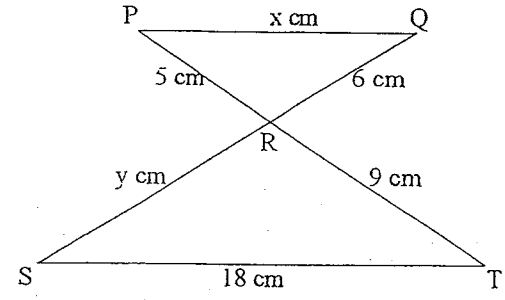
(b) _____ [1]

5 In the figure below , ΔPQR is similar to ΔTSR . Given that PR=5 cm , QR=6 cm , ST= 18 cm and TR= 9 cm , calculate the length

(i) PQ ,

(ii) SR .

[Solution]



ΔPQR is similar to ΔTSR

x/18 = 6/y = 5/9 (ratios of corr. sides of Δ)

x/18 = 5/9 -------M1 => x = 10 cm ----------A1

6/y = 5/9 --------M1

=> y = 10.8 cm or 10 4/5 cm ----------A1

Ans: (a) x = 10 cm [2]

(b) y = 10.8 cm [2]

6 Expand each of the following:

(a) $(3x - 5y)^2$ (b) $(2x + y)(x - 5y)$

(c) $(2x - 5y)(3x + y) + 5y(4x + y)$

[Solution]

(a) $(3x - 5y)^2$

$= 9x^2 - 30xy + 25y^2$ ---------- A1

(b) $(2x + y)(x - 5y)$

$2x^2 - 10xy + xy - 5y^2$ ---------- M1

$= 2x^2 - 9xy - 5y^2$ ---------- A1

(c) $(2x - 5y)(3x + y) + 5y(4x + y)$

$= 6x^2 + 2xy - 15xy - 5y^2 + 20xy + 5y^2$ --------M1

$= 6x^2 + 7xy$ ---------- A1

Ans: (a) $9x^2 - 30xy + 25y^2$ [2]

(b) $2x^2 - 9xy - 5y^2$ [2]

(c) $6x^2 + 7xy$ [2]

7 Factorise the following expressions completely

(a) $144x^2 - 25y^2$

(b) $5x^2 - 11x - 12$

[Solution]

(a) $144x^2 - 25y^2$
$= (12x)^2 - (5y)^2$ ----------M1
$= (12x + 5y)(12x - 5y)$ ---------- A1

(b) $5x^2 - 11x - 12$
$= (5x + 4)(x - 3)$ ---------- A2

Ans: (a) $(12x + 5y)(12x - 5y)$ [2]

(b) $(5x + 4)(x - 3)$ [2]

8 Evaluate the following by factorization without using calculators.

(a) $267^2 - 233^2$

(b) $32.6 \times 37 + 63 \times 32.6$

[Solution]

(a) $267^2 - 233^2$
$= (267 + 233)(267 - 233)$ ---------- M1
$= (500)(34)$
$= 17\,000$ ---------- A1

(b) $32.6 \times 37 + 63 \times 32.6$
$= 32.6(37 + 63)$ ----------M1
$= 32.6(100)$
$= 3\,260$ ---------- A1

Ans: (a) 17 000 [2]

(b) 3 260 [2]

9 Given that $x^2 - y^2 = 64$ and $x + y = 4$. Calculate the values of

(a) $x - y$,

(b) $(2x - 2y)^2$.

[Solution]

(a) $x^2 - y^2 = 64$
$\Rightarrow (x + y)(x - y) = 64$ ------ M1
$\Rightarrow 4(x - y) = 64$
$\Rightarrow (x - y) = 16$ ---------- A1

(b) $(2x - 2y)^2$.
$= [2(x - y)]^2$ ---------- M1
$= [2 \times 16]^2$
$= [32]^2$
$= 1\,024$ ---------- A1

Ans: (a) $(x - y) = 16$ [2]

(b) 1 024 [2]

10 Simplify each of the following expressions:

(a) $\dfrac{(7x^2y)^2z^4}{21yz}$ (b) $\dfrac{8x^2-16xy}{5x-10y}$

[Solution]

(a) $\dfrac{(7x^2y)^2z^4}{21yz}$

$= \dfrac{49x^4y^2z^4}{21yz}$ --------- M1

$= \dfrac{7x^4y\,z^3}{3}$ --------- A1

(b) $\dfrac{8x^2-16xy}{5x-10y}$

$= \dfrac{8x(x-2y)}{5(x-2y)}$ ---------- M1

$= \dfrac{8x}{5}$ ---------- A1

Ans: (a) $\dfrac{7x^4y\,z^3}{3}$ [2]

(b) $\dfrac{8x}{5}$ or $\dfrac{3}{5}x$ [2]

11 Simplify each of the following expressions:

(a) $\dfrac{x^2-25}{5-x}$ (b) $\dfrac{4x^2-y^2}{12x^2-4xy-y^2}$

[Solution]

(a) $\dfrac{x^2-25}{5-x}$

$= \dfrac{(x+5)(x-5)}{-(x-5)}$ ---------- M1

$= -(x+5)$ ---------- A1

(b) $\dfrac{4x^2-y^2}{12x^2-4xy-y^2}$

$= \dfrac{(2x+y)(2x-y)}{(6x+y)(2x-y)}$ ------M1 ------M1

$= \dfrac{2x+y}{6x+y}$ ---------- A1

Ans: (a) $-(x+5)$ [2]

(b) $\dfrac{2x+y}{6x+y}$ [3]

12 Simplify each of the following , giving your answers in the lowest terms.

(a) $\frac{6q^2}{21rp^2} x \frac{3pr^3}{2q} \div \frac{3r^2}{7pq}$ (b) $\frac{(p+2)(p+1)}{p+4} \div \frac{2p^2+p-1}{(2p-1)(p+4)}$

[Solution]

(a) $\frac{6q^2}{21rp^2} x \frac{3pr^3}{2q} \div \frac{3r^2}{7pq}$

$= \frac{6q^2}{21rp^2} x \frac{3pr^3}{2q} x \frac{3r^2}{7pq}$ ------ M1

$= q^2$ ---------- A1

(b) $\frac{(p+2)(p+1)}{p+4} x \frac{(2p-1)(p+4)}{2p^2+p-1}$ ---------- M1

$= \frac{(p+1)(p+2)}{(p+4)} x \frac{(2p-1)(p+4)}{(2p-1)(p+1)}$ ---------- M1

$= p + 2$ ---------- A1

Ans: (a) q^2 [2]

(b) $p+2$ [3]

13 Simplify each of the following as a single fraction.

(a) $\frac{2}{(x-3)} - \frac{1}{(2x-5)}$ (b) $\frac{3}{2(x-1)} - \frac{x+2}{(x^2-1)}$

[Solution]

(a) $\frac{2}{(x-3)} - \frac{1}{(2x-5)}$

$= \frac{2(2x-5)-1(x-3)}{(x-3)(2x-5)}$ ------- M1 for LCM

$= \frac{4x-10-x+3}{(x-3)(2x-5)}$ ---------- M1

$= \frac{3x-7}{(x-3)(2x-5)}$ -------- A1

(b) $\frac{3}{2(x-1)} - \frac{x+2}{(x^2-1)}$

$= \frac{3}{2(x-1)} - \frac{x+2}{(x+1)(x-1)}$

$= \frac{3(x+1)-2(x+2)}{2(x-1)(x+1)}$ --------- M1 for LCM

$= \frac{3x+3-2x-4}{2(x-1)(x+1)}$ ---------- M1

$= \frac{(x-1)}{2(x-1)(x+1)}$

$= \frac{1}{2(x+1)}$ ---------- A1

Ans: (a) $\frac{3x-7}{(x-3)(2x-5)}$ [3]

(b) $\frac{1}{2(x+1)}$ [3]

14 Given that $x = \frac{5y+3}{y+5}$, express y in terms of x.

[Solution]

$x = \frac{5y+3}{y+5}$ => x (y + 5) = 5y + 3 --------- M1

=> xy + 5x = 5y + 3

=> xy – 5y = - 5x + 3 or 5x – 3 = 5y – xy

=> y (x – 5) = - 5x + 3 ----- M1 5x – 3 = y (5 – x)

=> $y = \frac{3-5x}{x-5}$ --------- A1 $y = \frac{5x-3}{5-x}$

Ans: $y = \frac{3-5x}{x-5}$ [3]

15 Given that $s = r^2 + rl$, make l the subject of the formula. Taking $\pi = 22/7$, find the value of l when s = 200 and r = 5. Give your answer correct to 3 significant figures.

[Solution]

$s = r^2 + rl$ => $rl = s - r^2$ ---------- M1

$l = (s - r^2) / r$ ----------A1

$l = (s - r^2) / r$

= [200 – (22/7) $(5)^2$] / (22/7) x 5 ---------- M1

= 7.73 ---------- A1

Ans: (formula) $l =$ $(s - r^2) / r$ [2]

$l =$ 7.73 [2]

16 Solve the following pair of simultaneous equations using the elimination or substitution method.
$3x - 4y = 10$, $5x + 7y = 3$ [6

[Solution]

Elimination Method:

$3x - 4y = 10$ --------(1)

$5x + 7y = 3$----------(2)

From Eq(1) x 5 : $15x - 20y = 50$ ----------(3)

Eq(2) x 3 : $15x + 21y = 9$ ----------(4)

---------- M1

From Eq(3) – Eq(4) : $-41y = 41$------- M1

$y = -1$ -------- B1

Subst $y = -1$ into Eq(1) : $3x - 4(-1) = 10$----- M1

$3x = 6$

$x = 2$ ------B1

Substitution Method:

$3x - 4y = 10$ --------(1)

$5x + 7y = 3$----------(2)

From Eq(1): $3x = 10 + 4y$

$x = \frac{10 + 4y}{3}$ ----------(3)

---------- M1

Subt $x = \frac{10 + 4y}{3}$ into Eq(2)

$5(\frac{10 + 4y}{3}) + 7y = 3$ -------M1

$y = -1$ ---------- B1

Subt $y = -1$ into Eq(3):

$x = \frac{10 + 4(-1)}{3}$ ---------- M1

$x = 2$ ---------- B1

Therefore the solution of this pair of simultaneous equations is $x = 2$ and $y = -1$. ---------- A1

Ans: $x =$ _____

$y =$ _____

17 Complete the following table of values for x and y for the equation $3x - 4y = 10$. [2]

x	- 2	0	4
y	-4	- 2.5	0.5

(a) Using a scale of 2 cm to 1 unit on both axes , plot the points and draw a straight line through the points.

Plot all points correctly ---------- P1 [1]

Join points using straight line --------- D1 [1]

Label equation and axes ---------- L1 [1]

(b) From the graph, find

(i) the value of y when $x = -1$, [1]

(ii) the value of x when $y = -2$. [1]

[Solution]

(i) when $x = -1$, $y = -3.25$ (± 0.1) ---------- A1

(ii) when $y = -2$, $x = 0.67$ (± 0.1) ---------- A1

18 (a) Solve the equation $2x^2 - 9x = 0$

[Solution]

$2x^2 - 9x = 0$

$\Rightarrow x(2x - 9) = 0$ ----------M1

$\Rightarrow$ Let $x = 0$ or $2x - 9 = 0$

--------- A1 $x = 4$ ---------- A1

Ans: (a) $x = 0$ or 4 [3]

(b) The equation $2x^2 + kx - 15 = 0$ has a solution of $x = 3$.

(i) Find k.

(ii) Find the other x value.

[Solution]

Subt. $x = 3$ into $2x^2 + kx - 15 = 0$

$\Rightarrow 2(3)^2 + k(3) - 15 = 0$ ---------- M1

$\Rightarrow$ $k = -1$ ---------- B1

Subt. $k = -1$ into $2x^2 + kx - 15 = 0$

$\Rightarrow 2x^2 - x - 15 = 0$

$\Rightarrow (2x + 5)(x - 3) = 0$ ---------- M1

let $2x + 5 = 0$ or $x - 3 = 0$

$x = -2$ or $x = 3$

The other solution of the equation is $x = -2$ ---------- A1

Ans: (i) $k = -1$ [2]

(ii) $x = -2$ [2]

19 Solve the equation $4(x-3)(x+1) = 5(x-3)$

[Solution]

$4(x-3)(x+1) = 5(x-3)$

$\Rightarrow 4x^2 - 8x - 12 = 5x - 15$ ----------M1

$\Rightarrow 4x^2 - 13x + 3 = 0$

$\Rightarrow (4x - 1)(x - 3) = 0$ ---------- M1

let $4x - 1 = 0$ or $x - 3 = 0$

$\Rightarrow$ $x =$ $x = 3$

-------- A1 ---------- A1

Ans : $x =$ or 3 [4]

20 A housewife finds that 5 cans of condensed milk and 3 jars of instant coffee cost \$27 while 12 cans of condensed milk and 5 jars of instant coffee cost \$49.40. Assume that the cost of each can of condensed milk is \$x and each jar of instant coffee is \$y , form a pair of simultaneous equations and hence find the total cost for 7 cans of condensed milk and 2 jars of instant coffee. [7]

[Solution]

$5x + 3y = 27$ ---------- (1) ----------M1

$12x + 5y = 49.4$ ---------- (2) ---------- M1

from (1) x 5: => $25x + 15y = 135$ ---------- (3)

---------- M1

from (2) x 3: => $36x + 5y = 148.2$ ----------(4)

Eq (4) – (3): => $11x = 13.2$

$x = 1.2$ ---------- B1

Subst. $x = 1.2$ into (1) :=> $5(1.2) + 3y = 27$

$3y = 21$

$y = 7$ ---------- B1

The total cost of 7 cans of condensed milk and 2 jars of instant coffee
= \$(7x + 2y) ---------- M1

= \$[7(1.2) + 2(7)]

= \$ 22.40 (Ans) ---------- A1

21 **Answer the whole of this question on a sheet of graph paper.**

The following is a table showing corresponding values of x and y which are related by the equation $y = -x^2 + 2x + 3$.

x	-3	-2	-1	0	1	2	3	4	5
y	-12	a	0	3	4	b	0	-5	-12

(a) Calculate the values of a and b . [2]

(b) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 2 units on the y-axis , draw the graph of $y = -x^2 + 2x + 3$ for $-3 \leq x \leq 5$. [4]

(c) Use your graph to find

(i) the value of y when $x = 1.8$, [1]

(ii) the values of x when $y = -2$, [2]

(iii) the equation of the line of symmetry for $y = -x^2 + 2x + 3$, [1]

(iv) the maximum value of y . [1]

[Solution]

(a) when $x = -2$, $y = -(-2)^2 + 2(-2) + 3$

$a = -5$ ----------A1

when $x = 2$, $y = -(2)^2 + 2(2) + 3$

$b = 3$ ---------- A1

(b) Label axes and graph correctly ----------L1

Plot points correctly ----------P2

Smooth curve ---------- D1

(c) (i) when $x = 1.8$, $y = 3.36 (\pm 0.1)$ ---------- A1

(ii) when $y = -2$, $x = -1.45$ or 3.45 (± 0.1) ---------- A2

(iii) the equation of the line of symmetry of $y = -x^2 + 2x + 3$ is $x = 1$.--------- A1

(iv) the maximum value of y is 4 ---------- A1

END OF PAPER